Measuring the behavioral component of financial fluctuations: an analysis based on the S&P 500.*

Massimiliano Caporin   Luca Corazzini   Michele Costola

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Abstract

We estimate a Bayesian, mixture model of financial decisions with two typesof agents: one rational (endowed with preferences that are compatible with subjective expected utility theory) and one behavioral (with either an S-shaped or a reverse-S-shaped utility function). Agents take investment decisions by ranking the alternative assets according to their performance measures. We estimate the evolution of the behavioral component over time by using monthly data on the constituents of the S&P 500 index from January 1962 to April 2012. Our results confirm the existence of a significant behavioral component, which is more likely to emerge during periods of recession. We find a strong correlation between the behavioral component and the VIX index, with the relationship being stronger under the specification based on the S-shaped utility function.

JEL-Classification: G01, G02, G11, G17, C58.

Keywords: investment decision, behavioral agents, mixture model, behavioral expectations.

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1 Introduction

The main assumption of the traditional theory of finance (LeRoy and Werner, 2000) is that, in taking their financial investment decisions, agents maximize a well-conformed utility function that satisfies the requirements of the Subjective Expected Utility Theory (SEUT).

As well known, the validity of this hypothesis has been strongly questioned for its inability to account for systematic empirical puzzles, such as persistent mispricing of assets and the existence of arbitrage opportunities in the financial market (Hirshleifer, 2001; Barberis and Thaler, 2003; Lamont and Thaler, 2003).[1] Moreover, there is a huge experimental literature documenting systematic violations of the SEUT assumptions in risky gamble decisions (for an extensive and comprehensive survey, see Starmer, 2000). Once admitted that the requirements of the SEUT were rather than innocuous to describe financial decisions, scholars have started to introduce novel behavioral assumptions on individual preferences in their models. Nowadays, the so-called behavioral finance has established as a powerful and meaningful approach for financial economists. An intriguing research question that is still open in the literature concerns how to isolate and measure the behavioral component of the financial market. Indeed, while the most of the studies seek to elaborate sophisticated non-standard theories to rationalize puzzling evidence, little has been done to measure the relative relevance, at the market level, of the alternative behavioral hypothesis with respect to the benchmark SEUT setting. The present paper tackles this empirical issue. In particular, by using monthly data on the five hundred components of the S&P 500 index from January 1962 to April 2012, we propose a Bayesian mixture approach to estimate the relative importance of behavioral choices in the financial market. As in standard heterogeneous agent settings (Zeeman, 2007; Grossman and Stiglitz, 1980; De Long et al., 2008), our underlying model assumes that, in any time period, the evolution of the financial market reflects the interplay between the investment choices made by two categories of non-strategic financial agents: one rational, endowed with a standard risk averse utility function that is coherent with the SEUT requirements, and the other characterized by some other set of behavioral preferences. Then, the relative weight of the behavioral choices is estimated within a Bayesian framework combining the view of both types of agents. Intuitively, in our framework

[1] The equity premium puzzle surely represents the most intriguing empirical inconsistency studied by financial economists: although stocks on average exhibit attractive risk-return performances, investors appear to demand a substantial risk premium in order to prefer this asset to other riskless investment opportunities.

we consider the situation of an hypothetical agent who makes investment decisions by blending both the rational and the behavioral evaluations. A specific parameter of the model captures the uncertainty about the rational priors and, indirectly, it expresses the importance attached to the behavioral component. This parameter represents the main object of our methodology and is estimated by maximizing the investment performances obtained by blending the rational and the behavioral evaluations at the single asset level. We thus measure the extent of the behavioral component in an indirect way, by determining the relevance that the hypothetical agent should have attached to the behavioral evaluations relative to the rational counterpart, to maximize the financial performances of her investment. In that way, the "relevance" parameter monitors the impact of behavioral choices on market fluctuations. Our methodology is grounded on traditional agent's design, but also presents three main features that are useful to analyze financial data with the aforementioned purpose. First, it does not impose any particular restriction on the utility function to be used in order to describe the preferences of the behavioral agents. Of all the new non-Expected Utility theories proposed as valid alternatives to the SEUT, Prospect Theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992) represents the most successful and intriguing approach. According to the original formulation, agents' preferences are described by an S-shaped value function that presents two main attributes: (a) agents perceive a monetary outcome as a gain or a loss relative to a reference point; (b) agents' risk attitude changes over the monetary outcomes such that they exhibit risk averse preferences in the gain domain while they are risk lover in the loss domain. As a first step, we set the Bayesian mixture by assuming that the behavioral agents are endowed with a S-shaped utility function. Nevertheless, in order to check for robustness and in line with recent empirical findings (see Tibiletti and Farinelli, 2003; Malmendier and Nagel, 2011; Guiso, Sapienza and Zingales, 2013), we also replicate our analysis by considering a reversed-S-shaped value function that is concave in the loss domain and convex in the gain domain. Second, the relative importance of the behavioral component is estimated in every period by using an optimizing approach that is based on performance measures. Performance measures have several advantages from an empirical point of view. First, they summarize into a single parameter the interplay between risk and return of the corresponding asset. Second, performance measures can be ordered in such a way that assets with higher measures are more performing. Third, in order to define the performance measure of an asset, the financial agent chooses the partition of the wealth between a riskless activity and the risky asset that maximizes her utility function (Pedersen and Satchell, 2002).In a capital allocation setting, both the rational and the behavioral agents make choices by maximizing a specific performance measure, directly related to the feature of their utility functions. The risk averse agents consider the Generalized Sharpe ratio (see Zakamouline and Koekebakker, 2009b), while the Z-ratio is adopted by the behavioral agents (Zakamouline, 2011). In order to make their investments decisions, in any time period, each of the two categories of agents first determine the performance measure associated with each of the five hundred constituents of the S&P 500, and then builds a ranking going from the most to the least performing asset. Given the rankings built on the basis of their utility functions, agents allocate their wealth on the set including the "best performing" assets. . Given this framework, the financial market is seen as a composite of the choices made by the two categories of agents. Therefore, we can imagine that the optimal choice in the market might be that associated with the investment decisions taken by an hypothetic/imaginary agent that blends the two rankings. This agent produces a mixture ranking that is built by conditioning, in a Bayesian setting, the prior ordering of the rational, risk-averse agents on that produced by the behavioral category. The mixture depends on a weighting factor that expresses the confidence on the rational priors, or, from an antithetic point of view, the relative weight of the behavioral views over the rational ones: the higher the value of the weighting factor, the higher the relevance of the behavioral choices in the aggregated measure is. In particular, in every period, the estimated value of the weighting factor is obtained by maximizing the cumulated return of hundred given number of most performing assets of the mixture ranking. Intuitively, the weighting factor captures the extent to which the agents on their whole, and therefore the financial market, should have moved from the ordering of the rational category to the ranking of the behavioral agents to maximize their financial performances. Third, as the estimating procedure is recursively applied period by period, it allows to study how the relative importance of behavioral choices evolves over time and whether there exists a linkage between its evolution and the financial/economic cycle. For instance, under the assumption that the behavioral agents are endowed with an S-shaped loss averse value function produces the intuitive prediction that the attitude of undertaking risky investments changes according to the fluctuations of the financial market. On the one hand, in periods of (financial and economic) recession, financial agents are attracted by more risky investments which might generate, with some positive probability, returns that compensate previous (observed) losses. On the other hand, in periods of expansion, financial agents are more reluctant to undertake a risky investment which might reduce, with some positive probability, previous (observed) capital gains. Our results confirm the existence of a substantial behavioral impact in the financial market fluctuations. Under both the behavioral functions abovementioned, the weighting factor is significantly greater than zero and, coherently with the intuitive predictions discussed above, reaches its peaks in proximity of periods of financial and economic crises. Moreover, compared to a standard setting in which all agents are assumed to be rational, we find that the average return of the best (one hundred) assets selected by the mixture specification is more correlated to the average return of two benchmark selections, S&P 500 and S&P 100. To make their investment decisions, financial agents build the rankings by using a substantial quantity of information on the past returns of the assets. In particular, we assume that, in order to define the performance measure of an asset in a period, an agent considers the distribution of its past returns in the previous 60 months. This is compatible with the idea that the performance measure defined by an agent in a period represents her best adaptive expectation on the future performance of the corresponding asset. Different periods can be clearly used but those have an impact on the evaluation of performance measures: shorter periods increase the variability of performance measures and thus amplify the uncertainty of the rankings. On the basis of the previous considerations, in order to assess whether our methodology provides insight to explain (some proxy of) the real expectations in the financial market, we study the relationship between the estimated weighting factor (time) series and the VIX (CBOE, 2003). Under both the behavioral utility functions, we find a significant, high correlation between the estimated weighting factors and the VIX, suggesting that the behavioral component can account for a substantial portion of financial expectations. Interestingly, compared to the S-shaped specification, the correlation between the VIX index and the estimated weighting factors drops substantially under the reverse-S-shaped value function.

2 Different agents in the market

In our framework there exist two types of agents which differ on the base of their utility function. These decision makers must choose their optimal allocation and do their evaluation in terms of performance measures at the single asset level. As we will discuss later, performance measures are related to the level of maximum expected utility provided by a given single asset, and, generally speaking[2], are functions of the moments of the risky assets returns distribution. The higher the performance measure, the higher the maximum expected utility provided to the investor. Given performance measures at the single asset level, the allocation choice of the agent is made by investing in a subset of the assets (a fraction of the investment universe), those with higher scores in the performance measures. The first type of agent which we consider might be equipped with the classical utility function coming from the expected utility theory. We thus refer in this case to the optimal choices of a *rational* agent. The chosen utility function, the power utility, belongs to the class of Constant Relative Risk Aversion (CRRA) utility functions. Notably, as proved by Zakamouline and Koekebakker (2009b), the CRRA utility functions lead to the identification of a performance measure which is coherent with market equilibrium. The utility function of the *rational* agent might thus defined as follows:

$$U(W) = \begin{cases} \frac{1}{\rho} W^{1-\rho}, \text{ if } \rho > 0, & , \rho \neq 1 \\ \ln W \text{ if } \rho = 1 \end{cases} \tag{1}$$

where W is the agent's wealth and ρ measures the degree of relative risk aversion.

The power utility function has been extensively used in empirical studies, some of those aiming at identifying the value of ρ. The results of Mehra and Prescott (1985) indicates a value around 30 to ensure consistency with the observed market equity premium. As reported in Zakamouline and Koekebakker (2009b), for high values of ρ, the relative preferences across the moments of the distributions are similar to those of Constant Absolute Risk Aversion (CARA) utility functions.

In this regards, for computational convenience, we consider the a CARA instead of a CRRA utility function. Namely, we associate the *rational* agents with a negative exponential utility,

$$U(W) = -e^{-\lambda W} \tag{2}$$

where λ represents the coefficient of risk aversion. Such a coefficient affects the concavity property of the utility function, which is also influenced by the wealth of the investor W. An example of the utility function is reported in Figure 1.

The second type of agent we consider is characterized by a behavioral utility function. In general, we define a behavioural investor as a decision maker that

[2]This concept is related to the maximum principle introduced by Pedersen and Satchell (2002)

discriminates an outcome above and below a reference point, i.e. gains versus losses. Consequently, the investor's utility function behaves differently in the domain of gains and in the one of losses with a kink at the reference point,

$$U(W) \begin{cases} U_+(W) \text{ if } W \geq W_0, \\ U_-(W) \text{ if } W < W_0. \end{cases} \tag{3}$$

where W_0 is the reference point while $U_+(W)$ and $U_+(W)$ are two functions associated with the domains of gains and losses, respectively. According to the domain considered, gains or losses, different type of risks might arise from this behavioral utility function. Recently, Zakamouline (2011) has proposed a generalized *behavioural utility function* characterized by a piecewise linear plus power utility function,

$$U(W) = \begin{cases} 1_+(W - W_0) \times (W - W_0) - (\gamma_+/\alpha)(W - W_0)^\alpha, & \text{if } W \geq W_0, \\ -\lambda(1_-(W_0 - W) \times (W - W_0) + (\gamma_-/\beta)(W_0 - W)^\beta), & \text{if } W < W_0, \end{cases} \tag{4}$$

where , $1_+(\cdot)$ and $1_-(\cdot)$ are the indicator functions in $\{0, 1\}$ which define the linear part of the utility, and assume unit value for positive or negative arguments, respectively, and zero otherwise. Moreover, γ_+ and γ_- are real numbers that affect the shape of the utility and, finally, the additional parameters $\lambda > 0$, $\alpha > 0$ and $\beta > 0$ are real numbers. The utility function is continuous and increasing in wealth, and with proved existence of the first and second derivatives with respect to the wealth of the investor W.

The two utility functions previously described are generally considered for the evaluation of optimal investment decisions, or for the construction of optimal allocations between the risky asset and the risk-free investment, or within a set of risky assets. In our framework, the agents have to allocate their wealth across a set of risky investments. However, the allocation choices made by the decision makers is performed in term of the expected utility provided by each single asset. Then, given those single asset expected utility, the agents rank assets and invest on the top performers. Consequently, we refer to a *single* risky asset choice instead of a portfolio decision/allocation where many different risky activities are jointly considered. In this way, we are allowed to compare the different evaluation of the two type of investors across the assets (the investment universe). In practice, we are interested in the rankings provided by the *rational* and *behavioral* utility functions. We now describe how we derive asset ranks starting from the expected utility.

The expected utility of investment i is given as the convex combination of the utilities associated with a collection of different and alternative outcomes x_i, each corresponding to the realization of a given state of the world. Each realization is weighted by its respective probability, leading to following characterization of expected utility

$$\mathbb{E}\left[U\left(X\right)\right] = \int u(x)f(x)dx, \tag{5}$$

where $f(x)$ is the probability density function associating to each state of the world a given probability. In this case the utility is expressed as a function of the risky asset X, to highlight their relations. However, the wealth of the investor, not explicitly appearing, is also playing a role. In fact, the wealth W is always allocated between a *risky* and a risk free asset.

According to the maximum principle, the performance measure is strictly related to the level of maximum expected utility originated by a given financial activity.[3] In fact, the higher is the value of the performance measure, the higher is the maximum expected utility provided to the investor.

The Mean–Variance by Markowitz (1952) is a particular case of the expected utility theory when the returns are normally distributed. In this case, the Sharpe Ratio is the optimal solution for the maximization of the expected utility (the CARA negative exponential utility function).
Let's consider a decision maker with wealth W at the begin of a period t_0. Moreover, a denotes the amount of wealth allocated in a risky asset, while $W - a$ is the wealth allocated in the riskfree asset r_f. At the end of the period t_1 the wealth of the investor will be,

$$\tilde{W} = a \times (1 + x) + (W - a) \times (1 + r_f) = a \times (x - r_f) + w \times (1 + r_f) \tag{6}$$

where x is the return provided by the risky asset. In this framework, the aim of the investor is to maximize the expected utility with respect to the amount invested in the risky asset, a. Hence, the optimal problem corresponds to a utility maximization with respect to a,

$$\max_a E[U(\tilde{w})]. \tag{7}$$

Given the CARA function and the Gaussianity assumption, the maximized expected utility will be

$$E[U^*(\tilde{W})] = E[-e^{-\lambda[a(x-r_f)+W(1+r_f)]}] = E[-e^{-\lambda[a(x-r_f)]} \times e^{-\lambda W(1+r_f)}] \tag{8}$$

[3]The axiomatization approach is an alternative method for defining the performance measure. See De Giorgi (2005) and Cherny and Madan (2009).

where the last term within parentheses is a deterministic quantity.
By setting $x_0 = W(1 + r_f)$ as in Zakamouline (2011), we can approximate the expected utility using Taylor's series,

$$E[U(\tilde{W})] = -1 + a\lambda E(x - r_f) - \frac{\lambda^2}{2} a^2 E(x - r_f)^2 + O(\tilde{W}). \tag{9}$$

From the first order condition (FOC),

$$\frac{\partial E[U(\tilde{w})]}{\partial a} = \lambda E(x - r_f) - \lambda^2 E(x - r_f)^2 a = 0 \tag{10}$$

we obtain the *Sharpe Ratio* as the quantity that maximize the expected utility function,

$$a^* = \frac{1}{\lambda}\frac{\mu - r_f}{\sigma^2} = \frac{1}{\lambda}\frac{\text{SR}}{\sigma}. \tag{11}$$

However, the Sharpe ratio begins to be biased both in the measurement of optimal allocations and in the ranking across a collection of assets when there is a departure from the normal distribution assumption for the risky asset returns. This has been empirically demonstrated in Gatfaoui (2009), among others. To overcome this issue and still remaining within the expected utility maximization framework, Zakamouline and Koekebakker (2009b), among others, suggested the introduction of a generalized Sharpe ratio. Such a quantity would be sensitive to higher order moments, and can be evaluated with a parametric or a non-parametric methodology. In the non-parametric estimation, by following the Hodges (1998) conjecture, Zakamouline and Koekebakker (2009b) derived what they called Generalized Sharpe Ratio, GSR.
Recall first the maximization of the expected utility,

$$E[U(\tilde{W})] = E\big[-e^{-\lambda(x - r_f)}\big] = \max_a \int_{-\infty}^{\infty} -e^{-\lambda a(x - r_f)} \hat{f}_h(x) dx \tag{12}$$

where $\hat{f}_h(x)$ is now the estimated kernel density function of the risky asset returns. We thus differ from the previous simplified framework as we do not impose a parametric form to the risky asset return density. The GSR is obtained by the numerical optimization of the expected utility, see Zakamouline and Koekebakker (2009b),

$$GSR = \sqrt{-2\log(-E[U(\tilde{w})]}, \tag{13}$$

where the argument of the log in (13) is defined in (12). Note that, by resorting to the GSR, all moments of the risky asset returns play a role, and we are thus not constraining ourselves to the evaluation of the mean and variance. Notably, the GSR approaches to the standard Sharpe ratio when the underlying distribution of the risky asset returns is close to the Gaussian. We consider GSR as the performance measure adopted by the rational investor to rank risky assets. The rational investor would prefer assets with higher GSR to assets with lower values of the performance measure.

We move now to the choices of the behavioural agent. In this case, the expected generalised behavioural utility function can be approximated by a function of the mean and of partial moments of distribution. In this regards, Zakamouline (2011) verifies that the optimal allocation of an agent depends from a ratio playing the same role of the GSR, that is, the performance measure which maximizes the utility function for the behavioural agent. The new ratio, called the Z-ratio, has been derived with the use of the maximum principle and under some conditions, see Zakamouline (2011) for further details. The Z-ratio is given as

$$Z_{\gamma_-,\gamma_+,\lambda,\beta} = \frac{E(x) - r_f - (1_-(W - W_0)\lambda - 1)LPM_1(x, r_f)}{\sqrt[\beta]{\gamma_+ UPM_\beta(x, r_f) + \lambda\gamma_- LPM_\beta(x, r_f)}}.$$

where LPM and UPM are, respectively, the lower and upper partial moments as defined by Fishburn(1977),

$$LPM_n(x, r) = \int_{-\infty}^{r} (r - x)^n dF_x(x),$$
$$UPM_n(x, r) = \int_{r}^{\infty} (x - r)^n dF_x(x),$$

where n is the order of the partial moment of x at a threshold level r, usually set at the risk free return, and $F_x(\cdot)$ is the cumulative distribution function of x. We stress we will assume that the behavioural agents rank the risky assets using the Z-ratio of each risky asset. Similarly to the GSR, higher values of the Z-ratio are preferred to lower values.

There is one additional element we must consider when analysing the choices of behavioural agents. The utility function proposed by Zakamouline (2011) allows the construction of different preferences or beliefs of the agents through the calibration of the its parameters. Therefore, the concavity and convexity in the domain of gains and losses can be shaped in different ways and can give rise to different choices.

In this regards, we decline the general behavioral utility function in order to obtain an S-shaped utility similar to the the utility function used in prospect and cumulative prospect theory by Kahneman and Tversky (1979).[4] The utility

[4]The best choice would be the classical utility function by (Kahneman and Tversky, 1979):

$$\begin{cases} (W - W_0)^\alpha \\ -\lambda(W_0 - W)^\beta \end{cases}$$

function we chose is reported in Figure 2 and corresponds to the following choices for the parameters: $\gamma_+ = 0.1$, $\gamma_- = -0.1$, $\lambda = 1.5$, and $\beta = \alpha = 2$. The differences between the use of this utility function with respect to the classical S-shaped of Kahneman and Tversky (1979) lies in the definition of loss aversion. In our version of the S-shaped utility function of the decision maker exhibits loss aversion in the sense of Köbberling and Wakker (2005). The loss aversion is defined around the reference point in a local sense,[5] that is, if we define the ratio

$$\lambda = \frac{U'(W_0-)}{U'(W_0+)}$$

where in the numerator we have the left derivative and in the denominator the right derivative, the individual exhibits loss aversion if λ is greater than one.This implies that the utility function is steeper in the domain of losses: losses loom larger than corresponding gains, (Kahneman and Tversky, 1979). The main feature of the S-shaped utility function is the concavity in the gains and the convexity in the losses. In fact, the decision maker is risk adverse in the outcome above the reference point, and risk seeker below.[6]

Up to this point, we moved from the expected utility to the derivation of a performance measure. In turn, the last quantity is used by both the rational and behavioural agents to rank assets. One element is still missing, and refers to the construction of optimal allocations. We can here assume that agents allocate their wealth across the assets with highest ranks, that is highest values of the performance measure. If the market includes K assets, we might assume that the rational (behavioural) investor allocates his wealth across the $M << K$ assets with highest value of the GSR (Z-ratio). When doing that, agents might determine the optimal weights of those K assets, or simply use naive criteria such as resorting to an equally weighted allocation scheme. Note that the last

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Nonetheless, Zakamouline and Koekebakker (2009a) shown that the existence of the solution and thus the Z-ratio requires $\beta > \alpha$ which implies the absence of loss aversion in the utility.

[5]While Kahneman and Tversky (1979) define the loss aversion in a global sense,

$$-U(W_0 - \Delta W) > U(W_0 + \Delta W), \quad \forall \Delta W > 0.$$

See Zakamouline and Koekebakker (2009a) for a detail explanation.

[6]One well known experiment from (Kahneman and Tversky, 1979) is the choice among two lotteries in two different settings with their related probabilities:

- ($6,000,25%), or ($4,000,25%;$2,000,25%)
- (-$6,000,25%), or (-$4,000,25%;-$2,000,25%)

In the first problem, most of the individuals in the experiment choose the second option while in the second they choose the first option. Clearly, the first setting represents a choice in the domain of gains while the second a choice in the domain of losses. This gives rise to the concavity (convexity) in gains (losses) of the S-shaped utility function.

choice would allow limiting the impact the estimation error and has been shown to be preferred over optimal weighting schemes by DeMiguel et al. (2009). In this work we assume that agents allocate their portfolio using equal weights across a (relatively) small number of assets.

3 The Market Model

As we discussed in the previous section, we assume that two types of agents are present in the market. However, we do not know which type is prevailing, neither, irrespectively of their number, which type of agent is affecting more the market fluctuations. Our objective is to determine the relevance or the impact of behavioural choices in the movements of risky asset returns. We propose to recover such a measure in an indirect fashion by starting from the presence of two types of agents. Under this assumption, the observed market behaviour is a blend of choices made by rational and behavioural agents. As a consequence, one intuitive way to recover the impact of behavioural elements is to blend the choices of rational and behavioural agents and estimate the blending parameter(s) in such a way that the combination of choices is as closer as possible to the observed market fluctuations. In the following, starting from this intuition, we present our approach for recovering the impact and relevance of behavioural beliefs in a financial market.

An investor equipped with the expected utility theory is usually considered the benchmark for the rational investor. Therefore, according to Zakamouline and Koekebakker (2009b), the generalized Sharpe Ratio may represent the measure used to evaluate the assets in terms of this utility function. For the behavioural investor, we consider the S-shaped utility function introduced by Kahneman and Tversky (1979). In this case, the Zakamouline and Koekebakker (2009a) $Z-$ratio drives assets evaluation.

One way of blending the choices of the two agents types is to resort to a Bayesian framework where one of the two agent's beliefs is considered a prior, while the other agent choices assume the role of additional conditioning information. As a result, the posterior will represent a composite of rational and behavioural elements. From a Bayesian perspective, we define the prior as the rational investor. Such a choice is purely subjective, but allows, in a limiting case, to obtain the rational choices as the market outcome. The conditioning component is thus represented by the behavioural investor. As the choices of the two types of agents are driven by performance measures, GSR and Z-ratio,

the blending of choices is made at the performance measure level.
We thus start by assuming that both performance measures are normally distributed centred on their mean. For a generic performance measure PM we have

$$PM \sim N(\mu_{PM}, \sigma^2_{PM}). \tag{14}$$

Therefore, for the prior it holds that

$$\mu_{GSR} = GSR(E(U^*(\tilde{W}))) + \epsilon, \qquad \epsilon \sim N(0, \sigma^2), \tag{15}$$

while for the conditional we have

$$\mu_{\mathbb{Z}} = \mathbb{Z}_{\gamma_-,\gamma_+,\lambda,\beta,}(E(U^*(\tilde{W}))) + \eta, \qquad \eta \sim N(0, \omega^2). \tag{16}$$

Note that both distributions have mean set to the optimal choice for the agent, that is the Generalized Sharpe Ratio and the Z-ratio derived from market data. Moreover, the distributions refer to the performance measures of a single asset, that is, we have a collection of distributions, two for each risky asset present in the market. Finally, to simplify the treatment, we also assume that innovations, ϵ and η, are independent. Note that, by introducing innovations in (15) and (16) we are allowing for the presence of estimation error in the two measures. Differently, the distributional hypothesis in (14) takes into account the fact that agents aim at evaluating the expected value of a performance measure.
In order to determine the relevance of behavioural and rational choices, we modify the density in (15) by adding a multiplicative factor τ to the dispersion, leading to

$$\mu_{GSR} = GSR(E(U^*(\tilde{W})))) + \epsilon, \qquad \epsilon \sim N(0, \tau\sigma^2), \tag{17}$$

The coefficient τ can be interpreted as the reliability or uncertainty of rational (prior) expectations. The higher the τ the less reliable (more uncertain) are the rational choices, and thus higher weight might be given to behavioural elements. Conversely, the close is τ to zero, the lower is the uncertainty. By construction, and given the τ affects a variance, this parameter can assume values in the domain $[0, \infty]$.

The aggregation of rational and behavioural performance measures in a Bayesian framework gives rise to a composite performance measure consistent with (14) where mean and variance have the following expressions:

$$\mu_p = \left[(\tau\sigma^2)^{-1} + \omega^{-2}\right]^{-1} \left[(\tau\sigma^2)^{-1} GSR + \omega^{-2}\mathbb{Z}_{\gamma_-\gamma_+,\lambda,\beta}\right] \tag{18}$$

and

$$\sigma^2_p = \left[(\tau\sigma^2)^{-1} + \omega^{-2}\right]^{-1}. \tag{19}$$

Now, the aggregate expected measure, namely μ_p might be considered as the quantity used, at the market level, to order or rank assets. As a consequence, we might determine the role of behavioural choices through the composite measure, by looking at the optimal allocation made by an agent which is deciding where to invest his wealth across a set of risky assets ordered according to (18). In this case, the allocations might be evaluated in terms of past performances, while the impact of behavioural beliefs is determined by estimating the optimal τ level within a specified criterion function.

As we already noticed, we take a simplified allocation choice and consider an equally weighted investment strategy. Therefore, past performances can be evaluated as the cumulated returns of an equally weighted portfolio in a given time window, that is

$$r_p = \frac{1}{m} \sum_{l=t-m+1}^{t} r_{p,l} \tag{20}$$

where $r_{p,l}$ is the time l return of the equally weighted portfolio and m represents the time range for the portfolio evaluation (from time $t-m+1$ to time t). The portfolio is formed by the best performing equities according to (18). Let us collect in the set $\mathcal{A}_t(\tau)$ the M best assets across the K included in the market. This index is a function of the parameter τ because, by changing τ the asset ranks will be affected. Moreover, the set is also a function of time, given that the impact of behavioural choices might change over time.[7] Therefore, portfolio returns are represented as

$$r_{p,l} = \frac{1}{M} \sum_{j \in \mathcal{A}_t(\tau)} r_{j,l} \tag{21}$$

where $r_{j,l}$ is the return of asset j at time l; we stress that the index j vary from 1 to K but only M values are included in the set $\mathcal{A}_t(\tau)$. Given the dependence on τ of the best performing asset set, the portfolio cumulated return in (20) is also a function of τ. The optimal choice of τ is determined by maximizing the portfolio returns, that is

$$\begin{aligned} &\max_{\tau} f(\tau) = \frac{1}{m} \sum_{l=t-m+1}^{t} r_{p,l} \\ &s.t. \quad r_{p,l} = \frac{1}{k} \sum_{j \in \mathcal{A}_t(\tau)} r_{j,l} \end{aligned} \tag{22}$$

[7]Note that, in order to to simplify the notation, we avoid adding a time subscripts to the parameter τ.

The optimal value τ^* provides the maximum cumulated return obtained by an agent investing on a subset of the risky assets traded in the market and taking decisions blending rational and behavioural choices. As a consequence, the estimated τ^* represents the relevance of behavioural choices, or, conversely, the reliability on the rational beliefs.

In fact, a high value of τ^* would imply that the rational investor should have correct her action towards a behavioural direction. On the opposite, a low value of τ^* would imply that the investor should have remained on her prior rational beliefs. The criterion function allows detecting which component, rational versus behavioural, had a larger influence on the market.

The proposed approach is intimately linked to the investment decisions taken following the model introduced by Black and Litterman (1992).[8] In fact, our Bayesian combination is exactly equivalent to the Black and Litterman model where the rational choices are the prior expectations on asset returns (the equilibrium returns) and the behavioural choices plays the same role of the analysts views. In our implementation, both the prior and the views are univariate. Moreover, the methodology for the evaluation of optimal choices when a subset of risky assets is selected from an investment universe, is similar to the one adopted in Billio et al. (2012), in the framework of determining a composite performance measure by weighted linear combination of standard performance indices.

4 Empirical Analysis

4.1 The S&P 500 in 1962-2012

Generally, the stock market represents one of the most sensitive indicator for the business cycle as pointed out by Siegel (1991). Moreover, using a bivariate model with two regimes Hamilton and Lin (1998) have found that economic recessions are the main factor which leads the fluctuations in the volatility of the stock returns. Therefore, a focus on equities might allow the derivation of relevant evidences on the relation between economic and financial cycles, and on their association with agent's behaviour.

Our reference market is given by the equities included in the S&P 500 index in the period January 1962 to April 2012. The S&P 500 is a stock market index

[8] See He and Litterman (2002) for a detailed explanation of the model

by Standard & Poor's based on the 500 leading companies traded in the US.[9] Consequently, we focus the analysis on the components of the S&P500 market index across time.

The series of interest, the prices of the equities included in the index, have been downloaded from CRSP/COMPUSTAT at a monthly frequency. Moreover, we recover the US 3-Month Treasury Bill rates as a proxy used for the risk free rate.

Figure 4 shows the log–level of the S&P500 for the considered period, the bands in the plot represents the financial crisis according to Kindleberger and Aliber (2005). Figure 5 reports the bands of economic recessions according to the National Bureau of Economic Research (NBER). Tables 1 and 2 report the timing of financial and economic crisis, respectively.

Looking at the plots, it is natural to observe a match between the local minima in the log-index and the bands for the financial crisis. There is also a correspondence in the economic recessions. For instance, during the recession in the 1969–70 (the post-Vietnam era) a lower peak is clearly observable in Figure 5. This supports the validity of the financial market as a reliable indicator for the state of the economy.[10]

Table 3 contains some descriptive statistics grouped by decades. The period 1991-2000 has shown a great expansion phase, as it can be seen on the average returns. On the contrary, the last period from 2000-2012 has been the lowest in term of average returns. The risk level of the last decade is comparable to those observed in the range 1971-1990 where oil market shocks and the black Monday took place.

4.2 The Model specification and Empirical Results

The model we propose has been applied on rolling windows of 60 monthly returns in order to take into account the time-varying structure of the returns series. To implement our model and estimate the optimal value $\tau^{\star}$, at a given time t we select from the 500 stocks included in the index only those with at least 60 observations. We thus exclude those with a limited history where the evaluation of both the rational and behavioural performance measures might be characterised by a too large uncertainty. The variances of the performance measures are obtained using a block bootstrap procedure, setting the block to

[9] Equities are included in the index on the basis of their market value. The index composition is regularly revised.

[10] Note that our study does not focus on real-time detection of changes in the economic and financial cycles but rather on the association between them and the impact of behavioural decisions in the financial market.

a dimension of 4. Such a choice allows preserving any form of temporal dependence across the returns.[11] Such a procedure is repeated for each point in time, excluding the first five years, 1962-1969, which are needed to initialize the computation. At the end, we obtain a time series of optimal values $\tau_t^\star$. Note that for each utility function we recover a different sequence $\tau_t^\star$. We do not index the sequences with the utility function as the latter will be always explicitly indicated in the text. This also allows maintaining a simplified notation.

We filtered the optimized τ^* using a local level model in a state space representation. This allows extracting the level's signal component preserving its time variation:[12]

$$\begin{cases} \tau_t^* = \mu_t + \epsilon_t, & \epsilon_t \sim N(0, \sigma_\epsilon^2) \\ \mu_{t+1} = \mu_t + \xi_t, & \xi_t \sim N(0, \sigma_\xi^2) \end{cases} \tag{23}$$

where μ_t is the unobserved level, ϵ_t is the observation disturbance and $\xi_{i,t}$ is the level disturbance a time t. Both disturbances are identically and independently distributed according to a Gaussian density function. The estimated hyper-parameters of the model, using the filtered τ_t^* from the S-shaped utility function, are $\hat{\epsilon}_{1,t} \sim N(0, .4547)$ and $\hat{\xi}_{1,t} \sim N(0, .001678)$.

Figure 7 shows μ_t (henceforth, the filtered τ_t^*) including the economic recession bands according NBER.

The first check we consider for the filtered τ_t^* refers to the evaluation of the significance of that quantity. To this end we perform a TOBIT regression on the filtered τ_t^*, by specifying the censored dependent variable in the model. We thus set the lower bound equal to zero and check if the constant is significantly different from zero in the model

$$\mu_t = c + \epsilon. \tag{24}$$

Table 4 reports the results for the regressions in decades and for the full sample. The filtered τ_t^* is statistically different from zero in all the sub-samples and in the full sample. Other descriptive statistics are also included in the table. Looking at the dynamic of the filtered τ_t^*, see figures 4 and 5, it clearly emerges that we have three local maxima that coincide with the three longest economic recessions. The first is the oil crisis which corresponds to the highest value of the filtered τ_t^*. The second is the energy crisis which began with the Iranian revolution. According to Labonte and Makinen (2002), one of the main reason

[11] The bootstrap procedure has been applied to the returns. The measures have been computed in each iteration and then the variances have been obtained on the cross section of simulated measures.

[12] We thus filter out the noise and focus on the signal. See Koopman et al. (2012) for further details on the local level model.

for this crisis was due to the FED's monetary policy for the inflation control. This energy crisis is often considered a "Double Dip" recession with the previous one (January 1980 - July 1980); we found an inflection point in correspondence of this crisis in the series. In this regard, we found a similar result with the crisis from December 1969 to November 1970.
The last two shortest recessions are very similar to each other: both at the beginning of a decade (early-80s and early-90s) and both of the same length of eight months. In these cases, our estimated behavioural factor does not provide any particular pattern.
Finally, the third largest recession in the considered period is the sub-prime crisis (2007-2009). It is worth noting that the level of the filtered τ_t^* after the recession starts to decay very slowly. Then it remains substantially high at the begin of the European sovereign debt crisis.
As we might expect, we find the local minima in correspondence of booming periods of the equity market index. For instance, the first minimum is located just before the early-80s crisis (in the 1978) and the other is located just before the subprime crisis.
In the 1991-2000 decade, the economy has experienced a period of a solid economic growth; we found a relative low dynamic of the filtered τ_t^*.
Figure 4 represents the estimated factor including the bands for the financial crisis according to Kindleberger and Aliber (2005). Naturally, financial and economics crisis are highly interrelated and interdependent. Except for the 1987 stock market crash, they just anticipate or follow each other in most of the cases. Looking at the crisis, it is clearly observable a local minimum in the estimated factor before the begin of the crisis and then a local maxima during the crisis. As reported in Table (4), the period 1971-1980 and the period 1981-1990 contain on average the highest value and the highest standard deviation for the filtered τ_t^*. Probably, this is due for the two recessions in each decade.

We then move to analyse the relation between the filtered τ_t^* and the financial market's systemic component. Each optimal value of τ_t^* is associated with a selection of equities, those included in the portfolio returns in (21) evaluated at the optimal value τ_t^*. If the mixed selection and the extrapolation of the τ_t^* comes from two types of investors (weighted by the estimated mixing factor) it reflects the systematic component in the market. Therefore, if this relation is present, it is reliable to assume the presence of the two types of decision makers in the market. As a consequence, the market returns should be explained by the portfolio returns of this selection, that is the portfolio returns in (21).

In this regards, we estimate the following model,

$$r_{m,t} = c + \beta r_{\tau,t} + e, \tag{25}$$

where r_m is the S&P 500's return and r_τ is the return of the aggregated selection according to the optimal τ_t^*. Opposite to the CAPM model, the market return represents the dependent variable in this model. That is, if we assume a rational and behavioral investor in the market, the selection coming from the mixture of these two agents should largely explain the market returns.
Hence, according to our assumption, the model should return an high value for β and a constant close to zero. In the estimation, we use the equally-weighted returns for the S&P 500 (the dependent variable) since the returns from the selection are defined by the equally-weighted method.
Table 5 reports the estimated regression. The constant represents the risk premium which is slightly positive but close to zero. Economically, the result is coherent to what we might expect. Moreover, a positive sign is consistent with the efficiency of the market portfolio as shown in Sharpe (1966) and Fama (1998), since the selection is a subset of the available assets in the market each period. The β is significant at the 1% confidence level and has a value close to 0.90.
We perform a comparison also with the S&P 100 which includes the one hundred most capitalized companies in the US market. In this case, we use the value-weighted return series for the S&P 100 because of the short length of the equally-weighted series. The series for the index has been downloaded from Datastream and is available from January 1973; results are reported in Table 6.

The β is significant at 1% confidence interval and has now a value of 0.78. The constant is not significant. A lower beta in this case is quite reasonable for the different underlying market. In fact, some of the selected assets might be included in the S&P500 but not in the S&P100. However, the risk premium is not statistically different from zero and the β captures an high level of the systematic risk.
The analysis is also performed considering the rational agent's selection provided by the generalized Sharpe ratio. If we expect a coexistence between the two agents, the GSR-selection should capture a lower systematic component of the market. That is, a lower beta in the estimated model (25). Table 7 reports the results for the regression with the S&P 500 equally-weighted returns and Table 8 reports the results for the S&P100. The β coefficients are 0.83 and 0.64 respectively. These results confirm that the selection provided by the aggregated measure reflects an higher systematic part of the market with respect to the

selection resulting by rational agent's utility function. Again, it reasonable to assume the two types of agents in the market given these results.

5 The behavioural component and the VIX

At this point, we want to test if the filtered τ_t^* explains part of the market's fear expectations. Consequently, we use our estimated variable as an explanatory variable of a market sentiment index.

In this regard, we consider the CBOE Volatility Index (VIX). The VIX is a stock market volatility index introduced in the 1993 on the Chicago Board Options Exchange (CBOE).[13] It is also called the investor's fear gauge, since it is considered a measure of market expectations in the short-term period on the S&P 500's market (Whaley, 2000). [14] Thus, we consider the VIX the most appropriate choice as dependent variable to test if the filtered τ_t^* explains part of the market expectations. In Figure 8 we plot the filtered τ^* and the VIX.

In the regression, we use the estimated volatility of the S&P 500 as a control variable for the contemporaneous volatility in the market.

We consider the following model for the VIX

$$VIX_t = c + \beta_1 \tau_t^* + \beta_2 h_t^{1/2} + \eta_t, \tag{26}$$

where a second control variable is included. The quantity $h_t^{1/2}$ is the estimated conditional for the S&P500 returns and represents a statistical expectation of the market volatility. In the model for the index returns we consider a generalized error distribution (GED) and fit the $APARCH(P,O,Q)$ model by Ding et al. (1993). We select the simplest specification setting all orders to 1. We thus fit a APARCH(1,1,1) model obtaining the following estimates:

$$\begin{cases} x_t = \underset{(.0051)}{.0019} + h_t^{1/2}\epsilon_t, \\ h_t^{\delta} = \underset{(.0250)}{.0256} + \underset{(.0206)}{.1035}\left(|\epsilon_{t-1}| - \underset{(.1815)}{.8519}\epsilon_{t-1}\right)^{\delta} + \underset{(.0473)}{.8015}\sigma_{t-1}^{\delta} \end{cases} \tag{27}$$

where $\hat{\delta} = \underset{(.2900)}{.5246}$ and $\hat{\kappa} = \underset{(.1960)}{1.6149}$. See Ding et al. (1993) for additional details on the model.

The regression's results for the equation (26) are reported in Table 9. Both the explanatory variables are significant at 1% level of confidence. The filtered τ_t^*

[13] See CBOE (2003).

[14] In the 2003, a new methodology for the volatility index has been proposed. It has been calculated on the S&P500 index instead of the S&P 100 index. The Black and Scholes (1973) model has been replaced by fair value of future variance.[15]

coefficient is positive and equal to 0.3106.
Consequently, the filtered τ_t^* explains part of the VIX which is not related to the pure market volatility. We are thus capturing a dynamic element which is directly related to the market expectations.
Looking at the utility functions, the difference on preferences among the agents arises in the domain of losses: it is concave in the CRRA utility and convex in the S-shaped utility function. This suggests that we should expect to observe a different behavior of the agents during the period of crisis. Thus, when there is high volatility in the market.
In this perspective, τ captures the divergence in behavior between the two agents which is likely to emerge during turbulent financial periods.

6 Robustness Check

To check the consistency of our framework, we consider a different behavioral utility function.
In particular, an utility function which behaves in the opposite way of the behavioral S-shaped utility. Therefore, we consider an inverse-S-shaped utility function with no loss-aversion that is concave in the domain of losses (risk adverse) and convex in the gains (risk seeking).
Thus, as robustness check, we replicate our analysis considering the performance measure underlying this inverse-S-shaped utility function: the ratio proposed by Tibiletti and Farinelli (2003). In the model, we define this utility function following Zakamouline and Koekebakker (2009b).[16]
The estimation for the local level model in equation (23) for the filtered τ_t^* are $\hat{\epsilon}_{2,t} \sim NID(0, .1080)$ and $\hat{\xi}_{2,t} \sim NID(0, .04454)$.
Table 10 reports the estimates in decades and for all the sample. Also in this case, the filtered τ_t^* for this utility function is statistically significant from zero in all the sub-samples and in the entire sample. The descriptive statistics for the estimated factor are reported in Table 10.
We check also if the selection captures the systematic part of the market. The model (25) is analysed with the S&P500 and the S&P100. The results are

[16] In order to obtain this utility function, the parameters are set to:

$$\begin{cases} \gamma_+ = -\alpha, \\ \gamma_- = \beta \\ 1_+ = 0 \\ 1_- = 0 \end{cases} \quad \begin{cases} \lambda = 1.5 \\ \alpha = 1.5 \\ \beta = 2. \end{cases}$$

very similar to the S-shaped utility function case and confirms that also in this case, the selection reflect a systematic component. The estimated models are reported in Table 11 and in Table 12.

The most interesting part is the analysis of the relationship with the VIX in model (26). The results of the estimation are reported in Table 13. In this case, we have a negative relationship with the filtered τ_t^* which is consistent to what we should expect looking at the results of the S-shaped utility function.

7 Conclusions

Identifying and measuring the behavioral component in financial investment decisions is an open question for economists. For instance, there is a vivid empirical literature that studies how the relationship between investment choices and agents' risk attitude is mediated by behavioral components and changes overtime with the economic cycle. Guiso, Sapienza and Zingales (2013) elicit risk preferences using hypothetical lotteries in a repeated survey of Italian bank's clients and find that risk aversion increases substantially after the 2008 financial crises. Similarly, in a controlled experiment involving professionals, Cohn, Fehr and Maréchal (2012) found that, compared to expansion phases, financial crises trigger negative emotions and diminish risk taking choices in incentivized lotteries. Together, these findings are difficult to reconcile with the predictions of the Prospect Theory as they suggest that risk aversion is countercyclical. In this paper, we contribute to this flourishing literature by proposing a Bayesian mixture model to estimate the relative weight of the behavioral component of the financial market with respect to the benchmark SEUT setting. Rather than relying on experimental and survey data and with the intent of reducing the relevance of sampling and measurement errors, our analysis is based on real financial data. In particular, we use monthly data on the five hundred components of the S&P 500 index from January 1962 to April 2012. In line with the abovementioned literature, we detect a significant and time varying behavioral component that reaches its peaks during economic and financial crises. However, alike these studies, when using the mixture model to account for financial expectations and investors' sentiments, we find estimates from the mixture model based on a S-shaped value function with pro-cyclical risk aversion to better correlate with the VIX index than those implied by a reverse-S-shaped specification with countercyclical risk aversion. Our results are robust to changes in both the parameterization of the two behavioral utility functions and the definition of the set of the relevant assets used in the estimation procedure. In addition, our methodology is very flexible and can be easily modified to adapt to alternative behavioral utility functions.

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Crisis	Start date	End Date
The 1973 Oil Crisis	29-Oct-73	03-Oct-74
The 1987 Stock Market Crash	19-Oct-87	30-Dec-88
The 2000 Dotcom Bubble Burst	10-Mar-00	16-Apr-01
The 2001-9-11 Terrorist Attack	11-Sep-01	09-Oct-02
The Subprime Crisis	03-Dec-07	09-Mar-09

Table 1: The table provides the crisis list for the U.S. market according Kindleberger and Aliber (2005)

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Economic Recessions		
Quarterly dates are in parentheses		*DURATION IN MONTHS*
December 1969(IV)	November 1970 (IV)	11
November 1973(IV)	March 1975 (I)	16
January 1980(I)	July 1980 (III)	6
July 1981(III)	November 1982 (IV)	16
July 1990(III)	March 1991(I)	8
March 2001(I)	November 2001 (IV)	8
December 2007 (IV)	June 2009 (II)	18

Table 2: The table provides the crisis list for the U.S. economic recessions according NBER available at http://www.nber.org/cycles.html.

Period	*1962-1970*	*1971-1980*	*1981-1990*	*1991-2000*	*2001-2012*	*All-Sample*
Mean	0.0035	0.0043	0.0086	0.0124	0.0015	0.0060
Std	0.0384	0.0457	0.0474	0.0385	0.0466	0.0437
Skewness	-0.2874	0.1588	-0.6839	-0.5130	-0.5711	-0.4108
Kurtosis	2.9520	4.2453	6.5393	4.4303	3.7890	4.7155
Min	-0.0905	-0.1193	-0.2176	-0.1458	-0.1694	-0.2176
Max	0.1016	0.1630	0.1318	0.1116	0.1077	0.1630

Table 3: Descriptive statistics for the S&P500 index returns for the period January 1962 - April 2012.

year	*1962-1970*	*1971-1980*	*1981-1990*	*1991-2000*	*2001-2012*	*All-Sample*
c	1.0038	1.3851	1.1955	1.0109	1.0388	1.1408
s.e	0.0665	0.1543	0.1236	0.0266	0.0762	0.0303
pValue	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Skewness	-0.0696	0.0503	0.5803	0.0002	0.0308	1.0816
Kurtosis	2.1024	1.8382	1.7605	1.9997	1.8031	3.2658
Min	0.8869	1.1180	1.0632	0.9636	0.9127	0.8869
Max	1.1178	1.6505	1.4258	1.0632	1.1619	1.6505

Table 4: Results for the TOBIT regression and the descriptive statistics for the filtered τ for the S-shaped utility function in different periods.

	Estimate	SE	tStat	pValue
(Intercept)	0.0037	0.0009	4.0066	0.0001
r_τ	0.9049	0.0175	51.7944	0.0000
R^2	0.8322			
$\bar{R}^2$	0.8319	F-test	2682.6638	0.0000

Table 5: Regression where the dependent variable is the S&P500 equally weighted return and the explicative variable is return from the selection of the aggregated measure according τ^* for each period.

	Estimate	SE	tStat	pValue
(Intercept)	-0.0001	0.0009	-0.1177	0.9063
r_τ	0.7830	0.0172	45.4440	0.0000
R^2	0.8146			
$\bar{R}^2$	0.8142	F-test	2065.1593	0.0000

Table 6: Regression where the dependent variable is the S&P100 value weighted return and the explicative variable is return from the selection of the aggregated measure according τ^* for each period.

	Estimate	SE	tStat	pValue
(Intercept)	0.0025	0.0007	3.4410	0.0006
r_{GSR}	0.8339	0.0121	68.9244	0.0000
R^2	0.8978			
$\bar{R}^2$	0.8976	F-test	4750.5689	0.0000

Table 7: Regression where the dependent variable is the S&P500 value weighted return and the explicative variable is return from the selection of the Generalized Sharpe Ratio for each period.

	Estimate	SE	tStat	pValue
(Intercept)	-0.0004	0.0011	-0.3958	0.6924
r_{GSR}	0.6476	0.0182	35.6680	0.0000
R^2	0.8108			
$\bar{R}^2$	0.8104	F-test	1272.2073	0.0000

Table 8: Regression where the dependent variable is the S&P100 value weighted return and the explicative variable is return from the selection of the Generalized Sharpe Ratio for each period.

	Estimated	Robust s.e	tStat	pValue	R_p^2
(Intercept)	-0.3022	0.0589	-5.1339	0.0000	
τ_t^*	0.3106	0.0621	4.9979	0.0000	0.0908
$h_t^{1/2}$	1.2459	0.0954	13.0639	0.0000	0.4070
R^2	0.5944				
$\bar{R}^2$	0.5913	F-test	194.15	0.0000	

Table 9: The filtered τ^* is from the behavioral utility function Type 1.

year	*1962-1970*	*1971-1980*	*1981-1990*	*1991-2000*	*2001-2012*	*All-Sample*
Mean	0.7781	1.7586	0.5576	0.3068	0.5613	0.7876
s.e	0.1039	0.1036	0.0304	0.0211	0.0270	0.0352
pValue	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Skewness	1.1849	0.2916	1.9085	0.8835	1.4800	2.0073
Kurtosis	2.8534	1.7187	6.4377	2.4280	5.2108	6.4150
Min	0.1879	0.1810	0.1422	0.0476	0.2026	0.0476
Max	2.4935	4.0520	1.6790	0.8266	1.6969	4.0520

Table 10: Results for the TOBIT regression and the descriptive statistics for the filtered τ for the inverse S-shaped utility function in different periods.

	Estimate	SE	tStat	pValue
(Intercept)	0.0035	0.0009	3.8538	0.0001
r_τ	0.9213	0.0174	52.8160	0.0000
R^2	0.8376			
$\bar{R}^2$	0.8373	F-test	2789.5345	0.0000

Table 11: Regression where the dependent variable is the S&P500 equally weighted return and the explicative variable is return from the selection of the aggregated measure according τ^* in type 2 utility function for each period.

	Estimate	SE	tStat	pValue
(Intercept)	-0.0002	0.0009	-0.2621	0.7933
r_τ	0.7951	0.0177	44.8735	0.0000
R^2	0.8108			
$\bar{R}^2$	0.8104	F-test	2013.6313	0.0000

Table 12: Regression where the dependent variable is the S&P500 equally weighted return and the explicative variable is return from the selection of the aggregated measure according τ^* in type 2 utility function for each period.

	Estimated	Robust s.e	tStat	pValue	R_p^2
(Intercept)	-0.0247	0.0134	-1.8486	0.0725	
τ_t^*	-0.0263	0.0121	-2.1693	0.0384	0.0164
$h_t^{1/2}$	1.6015	0.1040	15.3994	0.0000	0.5119
R^2	0.5612				
$\bar{R}^2$	0.5579	F-test	169.45	0.0000	

Table 13: The filtered τ^* is from the behavioral utility function Type 2.

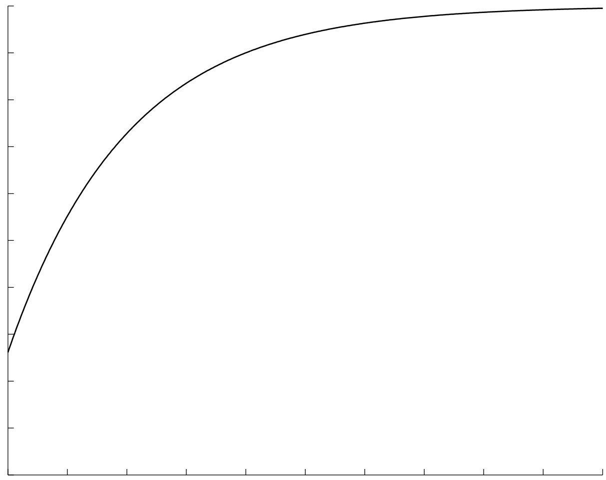

Figure 1: Negative exponential utility function with constant absolute risk aversion (CARA). λ is set equal to 1.5.

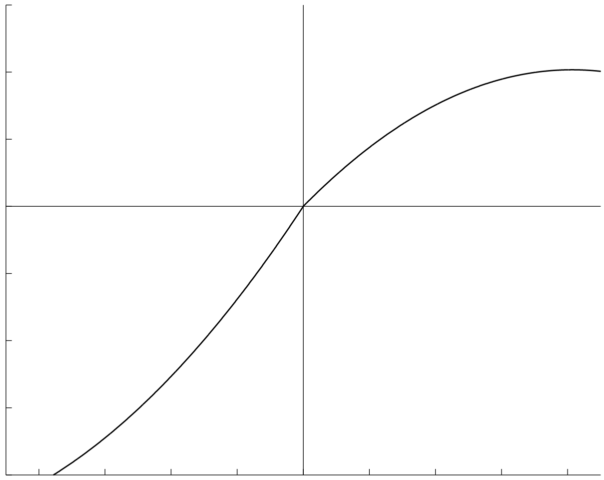

Figure 2: Type 1: Behavioral utility function similar to Kahneman and Tversky (1979).

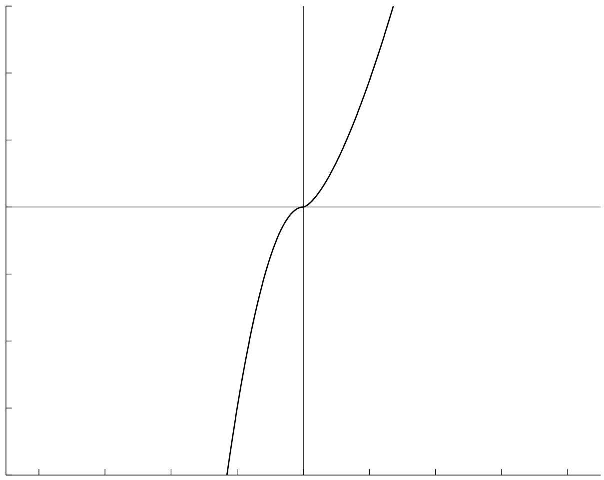

Figure 3: Type 2: Behavioral utility function concave on the domain of losses and convex in the domain of gains.

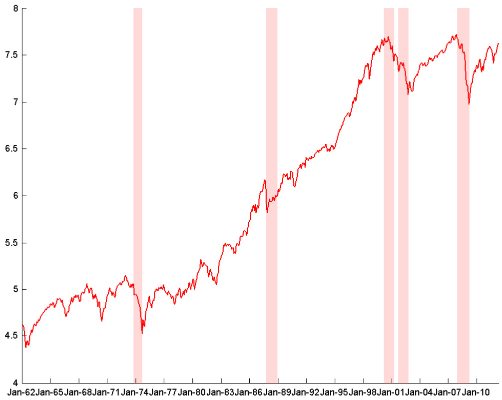

Figure 4: Log–level of the S&P500 index from January 1962 to April 2012 with bands for financial crisis. Source: Kindleberger and Aliber (2005)).

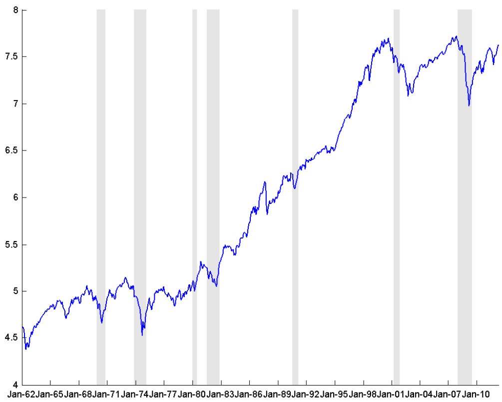

Figure 5: Log–level of the S&P500 index from January 1962 to April 2012 with bands for Economic Recessions. Source: NBER.

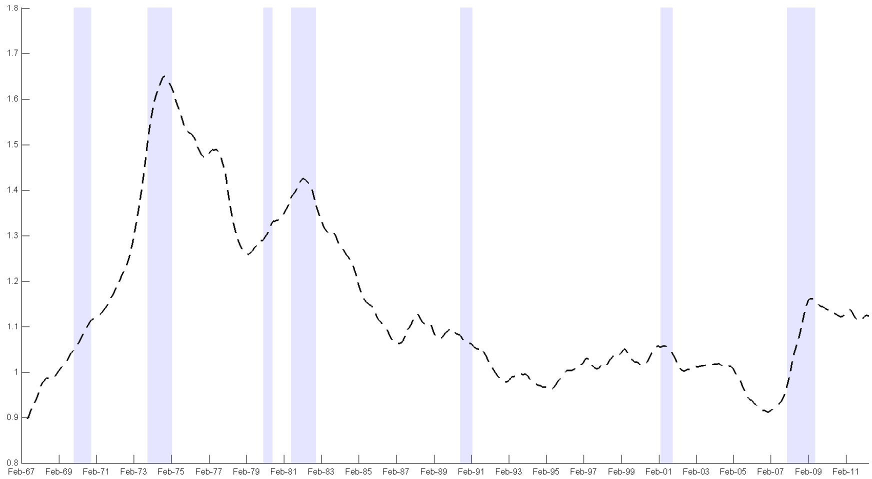

Figure 6: The filtered τ^*. The bands represent the Economic Recessions according NBER.

Figure 7: The filtered τ^*. The bands represent the Financial Crisis in the US based on Kindleberger and Aliber (2005).

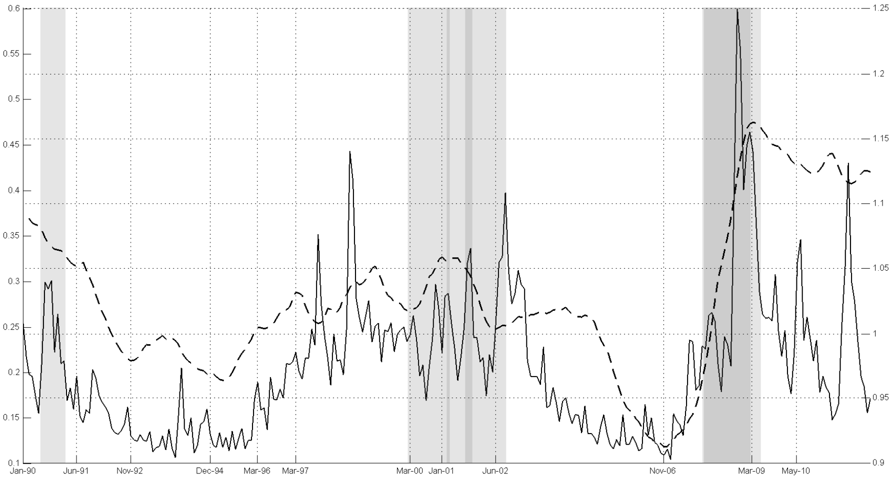

Figure 8: Vix (solid) and the filtered τ^* (dotted) from the utility function Type 1.

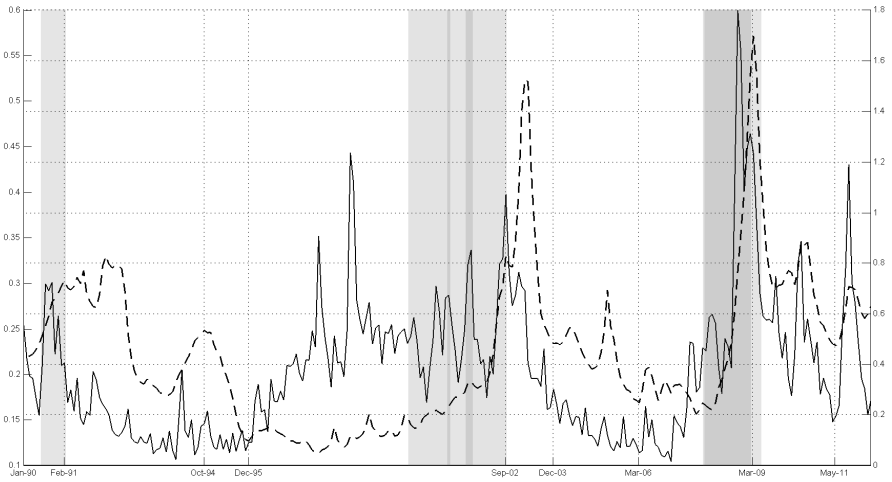

Figure 9: Vix (solid) and the filtered τ^* (dotted) from the utility function Type 2.

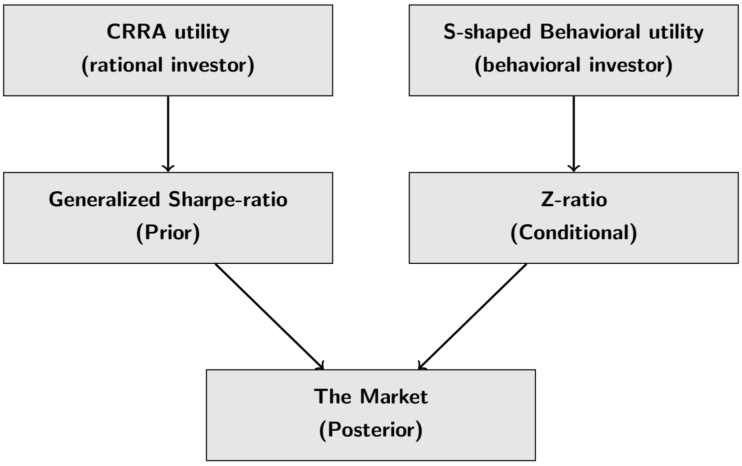

Figure 10: The Bayesian model which combines the two perspectives of the agents.